54 **a.** $(\vec{u}+2\vec{v})\cdot(5\vec{u}-\vec{v}) = 5\vec{u}^2 + 9\vec{u}\cdot\vec{v} - 2\vec{v}^2 = 5\times 4^2 + 9\times 5 - 2\times 2^2 = 117.$

b. $(\vec{u}+\vec{v})^2 = \vec{u}^2 + 2\vec{u}\cdot\vec{v} + \vec{v}^2 = 4^2 + 2\times 5 + 2^2 = 30.$

57 **a.** $\overrightarrow{AB}\cdot\overrightarrow{AC} = \frac{1}{2}(AB^2 + AC^2 - BC^2) = \frac{1}{2}(4^2 + 7^2 - 5^2) = 20.$

b. $\overrightarrow{AB}\cdot\overrightarrow{AC} = \frac{1}{2}(AB^2 + AC^2 - BC^2) = \frac{1}{2}(3^2 + 4^2 - 3^2) = 8.$

c. $\overrightarrow{AB}\cdot\overrightarrow{AC} = \frac{1}{2}(AB^2 + AC^2 - BC^2) = \frac{1}{2}(6^2 + 10^2 - (10^2 - 6^2)) = 36.$

En effet, $BC^2 = AC^2 - AB^2 = 10^2 - 6^2$ d'après le théorème de Pythagore.

d. $\overrightarrow{AB}\cdot\overrightarrow{AC} = \overrightarrow{AB}\cdot\overrightarrow{BD} = -\overrightarrow{BA}\cdot\overrightarrow{BD} = -\frac{1}{2}(BA^2 + BD^2 - AD^2) = -\frac{1}{2}(5^2 + 4^2 - 7^2) = 4.$

58 **a.** $\vec{u}\cdot\vec{v} = 1\times 1 + (-3)\times(-1) = 4.$

b. $\vec{u}\cdot(-4\vec{v}) = -4\vec{u}\cdot\vec{v} = -4\times 4 = -16.$

c. $-\vec{u}\cdot(2\vec{v}) = -2\vec{u}\cdot\vec{v} = -2\times 4 = -8.$

d. $(\vec{u}+\vec{v})\cdot(\vec{u}-\vec{v}) = \vec{u}^2 - \vec{v}^2 = (1^2 + (-3)^2) - (1^2 + (-1)^2) = 8.$

61 **a.** $\overrightarrow{AB}\cdot\overrightarrow{AC} = AB\times AC\times\cos\left(\overrightarrow{AB},\overrightarrow{AC}\right) = 4\times 5\times\cos(40°) \approx 15{,}32.$

b. $\overrightarrow{AB}\cdot\overrightarrow{AC} = \frac{1}{2}(AB^2 + AC^2 - BC^2) = \frac{1}{2}(4^2 + 6^2 - 3^2) = 21{,}5.$

c. En notant a = AB, $\overrightarrow{AB}\cdot\overrightarrow{AC} = a \times \frac{6}{2} = 3a.$

d. AD = DC, donc $2AD^2 = 3$ d'où $AD = \sqrt{\frac{3}{2}}.$

$\overrightarrow{AB}\cdot\overrightarrow{AC} = \overrightarrow{AB}\cdot\left(\overrightarrow{AD}+\overrightarrow{DC}\right) = 0 + 5\times\sqrt{\frac{3}{2}} = 5\sqrt{\frac{3}{2}}.$

e. $\overrightarrow{AB}\cdot\overrightarrow{AC} = \overrightarrow{AB}\cdot\left(\overrightarrow{AD}+\overrightarrow{DC}\right) = \overrightarrow{AB}\cdot\overrightarrow{AD} + \overrightarrow{AB}\cdot\overrightarrow{DC} = AB\times AH + AB^2 = 5\times 1 + 5^2 = 30.$

f. $\overrightarrow{AB}\cdot\overrightarrow{AC} = AB\times AC\times\cos\left(\overrightarrow{AB},\overrightarrow{AC}\right) = 7\times 1\times\cos\left(\frac{2\pi}{3}+\pi\right) = 7\times 1\times\frac{1}{2} = 3{,}5.$